Simple and Compound Interest

Grade 10 Mathematics

Australian Curriculum

Learning Objectives

By the end of this lesson, you should be able to:

- Calculate simple interest.
- Calculate the final amount using simple interest.
- Find the interest rate or time when other values are known.
- Calculate compound interest.
- Work with different compounding periods.
- Compare simple and compound interest.
- Solve financial mathematics problems in real-life contexts.

Key Vocabulary

- **Principal (P):** The original amount invested or borrowed.
- **Interest (I):** The money earned or charged.
- **Rate (r):** The percentage interest rate per year.
- **Time (t):** The length of the investment or loan.
- **Amount (A):** The total value after interest is included.

Remember: Convert percentages to decimals.

$$5\% = 0.05 \qquad 8.5\% = 0.085 \qquad 11\% = 0.11$$

Simple Interest

With simple interest, interest is calculated only on the original principal.

$$\boxed{I = Prt}$$

where:

$P =$ principal, $r =$ annual rate as a decimal, $t =$ time in years

The final amount is:

$$\boxed{A = P + I}$$

Simple Interest — Example

If $P = \$1000$, $r = 6\%$, and $t = 3$ years:

$$I = (1000)(0.06)(3) = \$180$$

$$A = 1000 + 180 = \boxed{\$1180}$$

Worked Example 1 — Simple Interest

Sam deposits **$1000** into a bank account.
The account pays **11% simple interest per annum**.
How much will Sam have after **2.5 years**?

Step 1: Identify the values $P = 1000,\ r = 0.11,\ t = 2.5$
Step 2: Calculate interest $I = Prt = (1000)(0.11)(2.5) = 275$
Step 3: Calculate the final amount

$$A = P + I = 1000 + 275 = \boxed{\$1275}$$

Worked Example 2 — Finding the Interest Rate

A student invests **$2000** for **3 years**.
They earn **$360** in simple interest.
What was the annual interest rate?

Start with $I = Prt$. Substitute:

$$360 = (2000)(r)(3)$$

$$360 = 6000r$$

$$r = \frac{360}{6000} = 0.06$$

Convert to a percentage: $0.06 = 6\%$, so

$$\boxed{r = 6\% \text{ p.a.}}$$

Worked Example 3 — Finding Time

A person invests **$1500** at **8% simple interest**.
They earn **$360** interest.
How long was the money invested?

$$I = Prt$$

$$360 = (1500)(0.08)t$$

$$360 = 120t$$

$$t = \frac{360}{120}$$

$$\boxed{t = 3 \text{ years}}$$

Compound Interest

With compound interest, interest is added to the account.
The next interest calculation is therefore based on the **new balance**.

If interest is compounded n times per year:

$$\boxed{A = P\left(1 + \frac{r}{n}\right)^{nt}}$$

For annual compounding:

$$n = 1$$

so:

$$\boxed{A = P(1 + r)^t}$$

Simple vs Compound Interest

	Simple Interest	**Compound Interest**
Interest calculated on	Original principal	Growing balance
Formula	$I = Prt$	$A = P(1 + r)^t$
Interest changes?	No	Usually increases
Growth	Linear	Exponential

Key idea:
With compound interest, you earn:

interest on interest

Worked Example 4 — Compound Interest

Oliver deposits **$2000** into an account paying **5% compound interest annually**. Calculate the amount after **3 years**.

Use $A = P(1 + r)^t$. Substitute:

$$A = 2000(1 + 0.05)^3$$

$$A = 2000(1.05)^3$$

$$A = 2000(1.157625)$$

$$A = \boxed{\$2315.25}$$

Interest earned: $2315.25 - 2000 = \boxed{\$315.25}$

Worked Example 5 — Quarterly Compounding

A student invests **$4000** at **6% p.a.**, compounded quarterly.
Calculate the value after **3 years**.

Quarterly means $n = 4$. Use $A = P\left(1 + \frac{r}{n}\right)^{nt}$:

$$A = 4000\left(1 + \frac{0.06}{4}\right)^{4(3)}$$

$$A = 4000(1.015)^{12}$$

$$A \approx 4000(1.195618)$$

$$A \approx \boxed{\$4782.47}$$

Compounding Periods

The value of n depends on how frequently interest is compounded.

Compounding	**n**
Annually	1
Half-yearly	2
Quarterly	4
Monthly	12
Daily	365

Important:
If the interest rate is annual, divide the rate by n and multiply the number of years by n.

Worked Example 6 — Comparing Investments

A student invests **\$2500** for **5 years**.

- Account A: 6% simple interest
- Account B: 6% compound interest annually

Which account gives the greater return?

Account A: $I = Prt = (2500)(0.06)(5) = 750$, so $A = 2500 + 750 = \boxed{\$3250}$

Account B: $A = 2500(1.06)^5 = \boxed{\$3345.56}$

Therefore $3345.56 - 3250 = \boxed{\$95.56}$ — **Account B is better by \$95.56.**

Practice Questions

Show all working.

1. Calculate the simple interest on \$800 at 5% p.a. for 3 years.
2. Calculate the final amount for \$1500 invested at 6.5% simple interest for 4 years.
3. \$2400 is invested at 4.8% simple interest for 2.5 years. Find the interest earned.
4. \$2000 earns \$540 simple interest over 3 years. Find the annual interest rate.
5. Calculate the value of \$2000 invested at 5% compound interest annually for 3 years.

Practice Questions — Compound Interest

6. \$4500 is invested at 4% compound interest annually for 5 years. Find the final amount.
7. \$3000 is invested at 6% compound interest annually for 4 years.
   Find:
   - the final amount;
   - the total interest earned.
8. \$4000 is invested at 6% p.a. compounded quarterly for 3 years.
9. \$5000 is invested at 4.8% p.a. compounded monthly for 3 years.
10. Compare \$2500 invested for 5 years at:
    - 6% simple interest;
    - 6% compound interest annually.

Challenge Questions

11. Emily has \$6000 to invest for 4 years.
    Investment A offers 5.5% simple interest.
    Investment B offers 5.5% compound interest annually.
    Which investment should Emily choose?
12. A student deposits \$5000 into an account earning 4.8% p.a. compounded monthly.
    Calculate the balance after 3 years.
13. Sam deposits \$1000 at 11% simple interest.
    - How much will Sam have after 2 years?
    - How much will Sam have after 2.5 years?
    - How much additional interest is earned during the extra 0.5 years?

Answers — Final Page (1–7)

1. $\boxed{\$120}$
2. $\boxed{\$1890}$
3. $\boxed{\$288}$
4. $\boxed{9\%}$
5. $\boxed{\$2315.25}$
6. $\boxed{\$5474.94}$
7. $\boxed{\$3787.43}$ final amount; $\boxed{\$787.43}$ interest

Answers — Final Page (8–13)

8. $\boxed{\$4782.47}$
9. $\boxed{\$5772.76}$
10. Simple: $\boxed{\$3250}$; Compound: $\boxed{\$3345.56}$
11. Investment B; approximately $\boxed{\$7432.95}$
12. $\boxed{\$5772.76}$
13. $\boxed{\$1220}$ after 2 years; $\boxed{\$1275}$ after 2.5 years; additional interest $\boxed{\$55}$

Great Work!

Remember:

Simple Interest: $I = Prt$

Compound Interest: $A = P\left(1 + \frac{r}{n}\right)^{nt}$